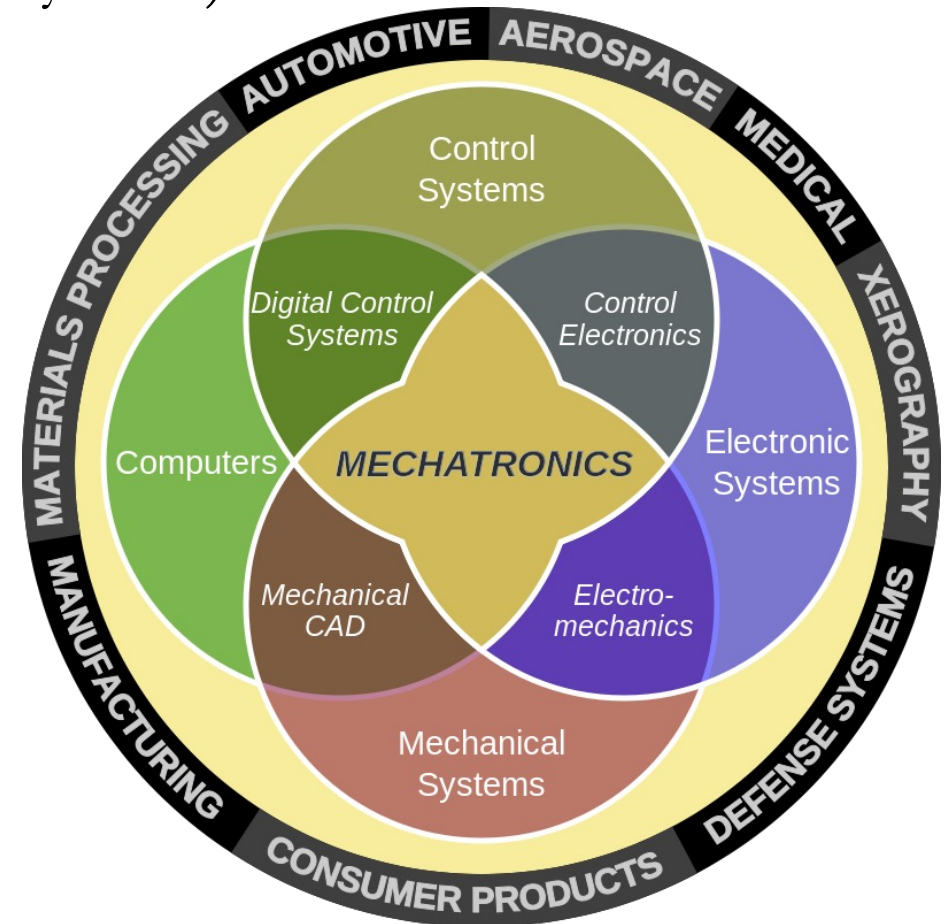# Mechatronic Modeling and Design with Applications in Robotics
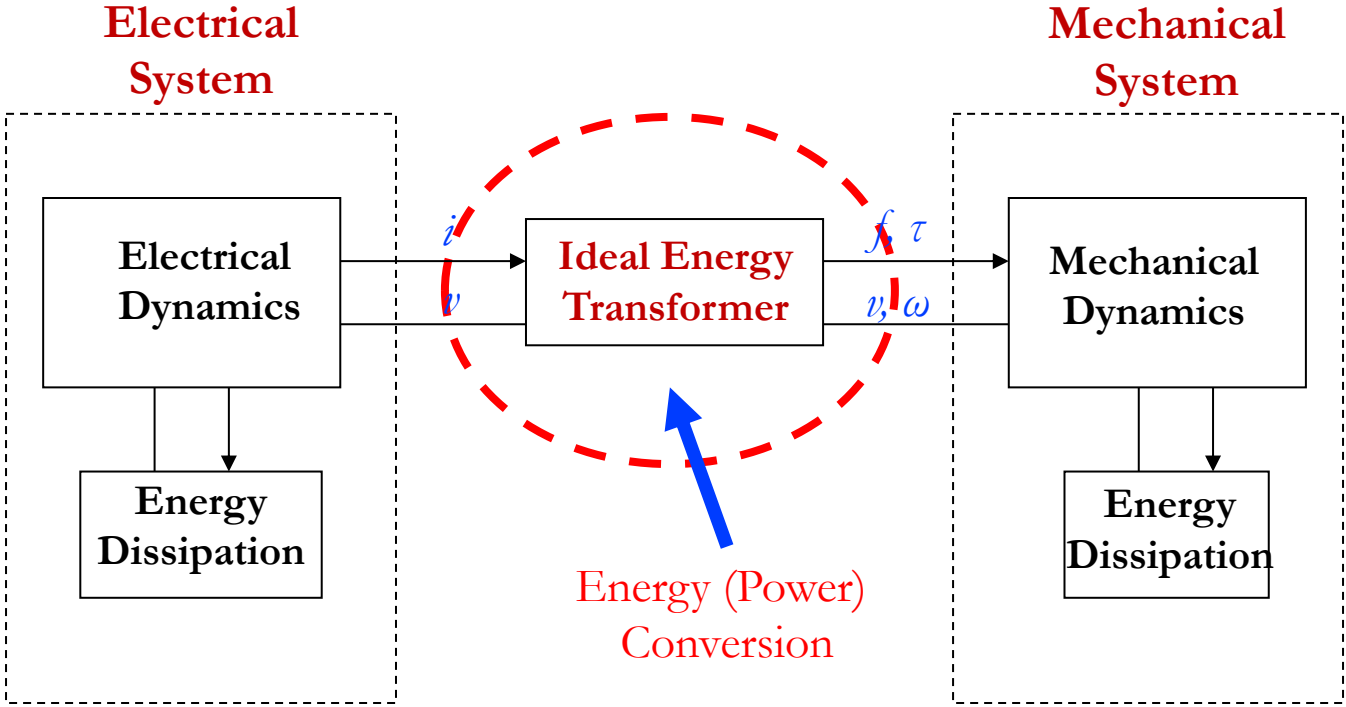
Basic Model Elements

The field of mechatronics primarily concerns the integration of mechanics and electronics.
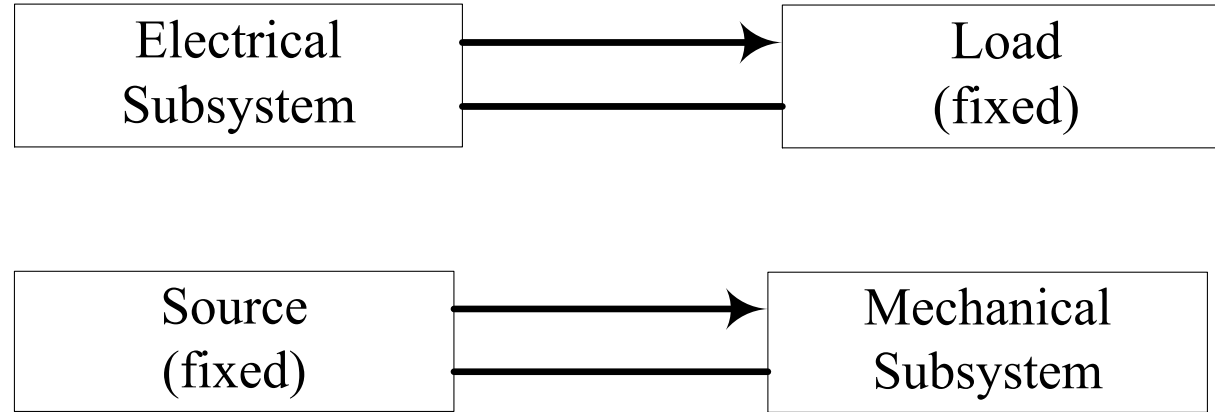(e.g., mechanical, fluid, thermal and electrical/electronic systems)

They can serve functions of
- Structural support
- Load bearing
- Mobility
- Transmission of motion and energy
- Actuation
- Manipulation
- Sensing
- Control

https://en.wikipedia.org/wiki/Mechatronics

Electrical System

Mechanical System

Electrical Dynamics

i

v

Ideal Energy Transformer

f, τ

v, ω

Mechanical Dynamics

Energy Dissipation

Energy Dissipation

Energy (Power) Conversion

An electromechanical system / mechatronic system

```
┌──────────────┐        ┌──────────────┐
│  Electrical  │ ──────▶│    Load      │
│  Subsystem   │ ───────│   (fixed)    │
└──────────────┘        └──────────────┘


┌──────────────┐        ┌──────────────┐
│    Source    │ ──────▶│  Mechanical  │
│   (fixed)    │ ───────│  Subsystem   │
└──────────────┘        └──────────────┘
```

❖ Energy (or Power)

❖ Bandwidth (e.g., Speed and Time Constant)

Required and needed in this course:
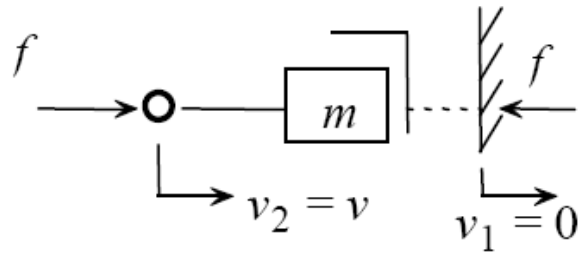
➢ Mechanical Components

➢ Electrical Elements

Should understand:
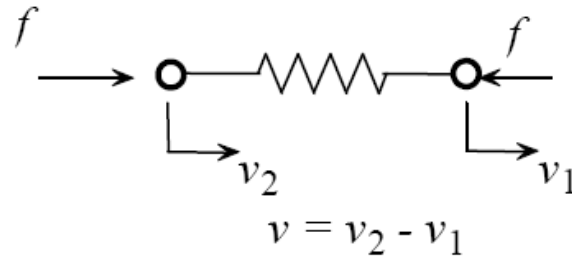
➢ Fluid Elements

➢ Thermal Elements

Across Variable: Varies Across Element (e.g., Velocity, Voltage, Temperature, Pressure)

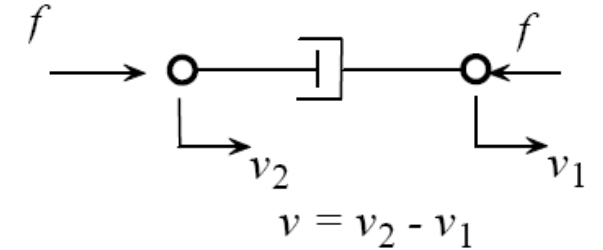**Through Variable:** Remains Unchanged Through Element (e.g., Force, Current, Heat Transfer Rate, Fluid Flow Rate)

Mass   **Spring**   **Damper**

Sources: Velocity and force/torque

Variables: Velocity (across variable) and force (through variable)

Mass (Inertia) Element (A-Type Element)

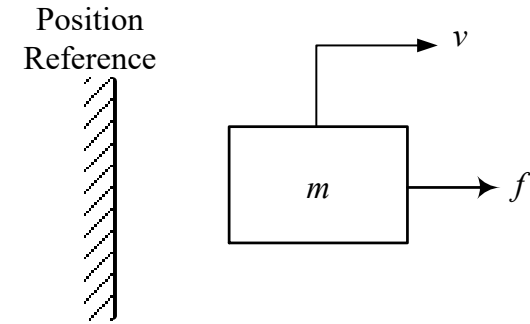**Constitutive Equation (Newton's 2nd Law):**

$$f = m\frac{dv}{dt}$$

where m = mass(inertia)

Power = $f\,v$ = rate of change of energy ➔

$$E = \int f v\,dt = \int m\frac{dv}{dt}v\,dt = \int mv\,dv$$

➔ Energy $E = \frac{1}{2}mv^2$ (Kinetic Energy) ➔ Energy storage element

➤ An inertia is an energy storage element (kinetic energy).

➤ Velocity (across variable) represents the state of an inertia element ➔ "A-Type Element"

Note: 1. Velocity at any t is completely determined from initial velocity and the applied force; 2. Energy of inertia element is represented by v along.

➤ Hence, v is a natural output (or response) variable for an inertia element, which can represent its dynamic state (i.e., state variable), and f is a natural input variable for an inertia element.

➤ Velocity across an inertia element cannot change instantaneously unless an infinite force is applied to it.

Spring (Stiffness) Element (T-Type Element)

Constitutive Equation (Hooke's Law):

$$\frac{df}{dt} = kv$$

where k=stiffness
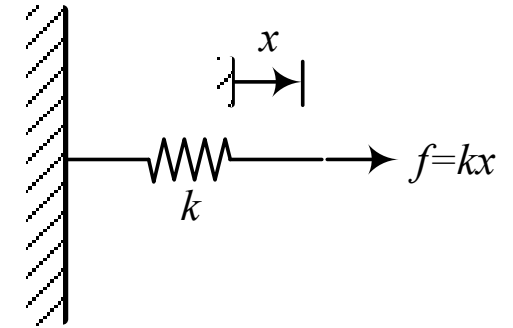
Note: Differentiated version of familiar force-deflection Hooke's law in order to use velocity (as for inertia element)

$$E = \int fvdt = \int f\frac{1}{k}df$$

➔ Energy $E = \frac{1}{2}\frac{f^2}{k}$ (Elastic potential energy)

➔ Energy storage element

➢ A spring (stiffness element) is an energy storage element (elastic potential energy).

➢ Force (through variable) represents state of spring element ➔ "T-Type Element".

Note: 1. Spring force of a spring at time t is completely determined from initial force and applied velocity; 2. Spring energy is represented by f alone.

➢ Force f is a natural output (response) variable, and v is a natural input variable for a stiffness element.

➢ Force through a stiffness element cannot change instantaneously unless an infinite velocity is applied to it.

Damping (Dissipation) Element (D-Type Element)

$f = bv$
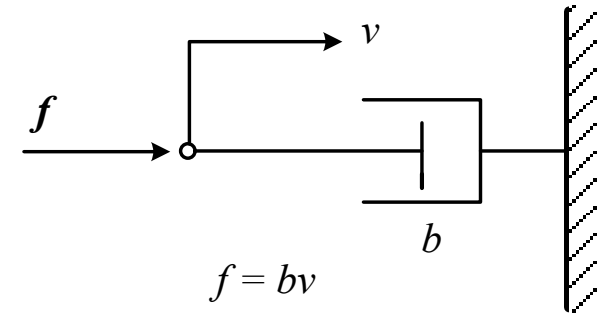
Constitutive Equation: $f = bv$

where b=damping constant (damping coefficient); for viscous damping

The power dissipated depending on the velocity v:

$$P = bv^2$$

➢ Mechanical damper is an energy dissipating element (D-Type Element).

➢ Either force f or velocity v may represent its state.

➢ No new state variable is defined by this element.

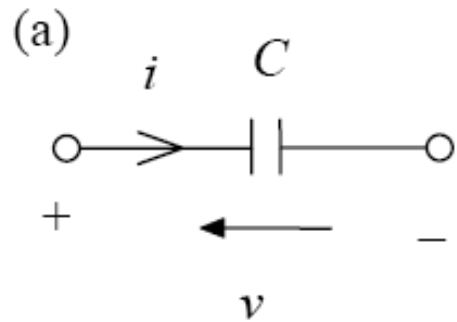Rotational Mass:

$$E = \frac{1}{2}I\omega^2$$

Torsional Spring:

$$E = \frac{1}{2}\frac{T^2}{k}$$
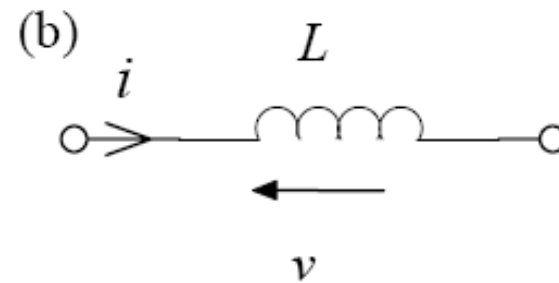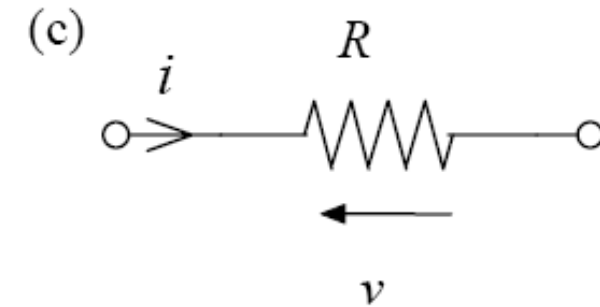
Rotary Damper:

$$P = c\omega^2$$

Capacitor   **Inductor**   **Resistor**

Sources: Voltage and current

Variables: Voltage (across variable) and current (through variable)

Variables: Voltage (across variable) and the current (through variable)
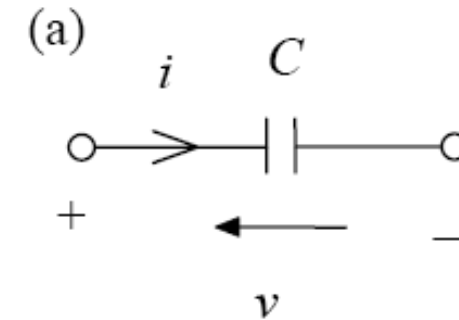
Capacitor Element (A-Type Element)

Constitutive Equation: $C \dfrac{dv}{dt} = i$

where C = capacitance

(a)

Power = iv ➔ Energy $E = \int ivdt = \int C\dfrac{dv}{dt}vdt = \int Cvdv$ ➔

Energy $E = \dfrac{1}{2}Cv^2$ (electrostatic energy) ➔ Energy storage element

➢ Voltage (across variable) is state variable for a capacitor ➔ "A-Type Element".

➢ Voltage is a natural output variable and current is a natural input variable for a capacitor.

➢ Voltage across a capacitor cannot change instantaneously unless an infinite current is applied.

Inductor Element (T-Type Element)

(b)

Constitutive Equation: $L \dfrac{di}{dt} = v$

where L = inductance

Energy $E = \dfrac{1}{2} L i^2$ (Electromagnetic energy)
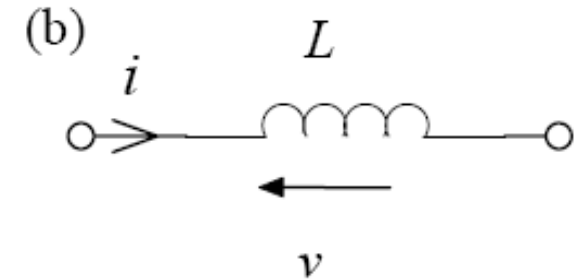
➢ Current (through variable) is state variable for an inductor ➔ "T-Type Element".

➢ Current is a natural output variable and voltage is a natural input variable for an inductor.

➢ Current through an inductor cannot change instantaneously unless an infinite voltage is applied.
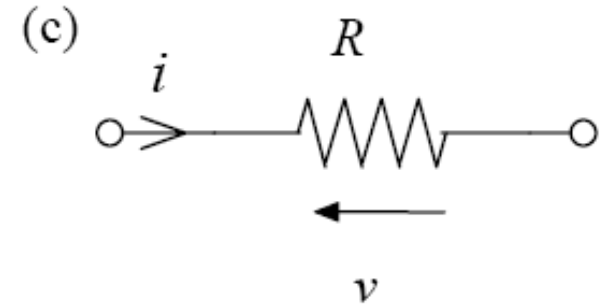
Resistor Element (D-Type Element)

Constitutive Equation: $v = Ri$ (Ohm's law)

where R = resistance

Observations:

1. This is an energy dissipating element (**D-Type Element**)

2. Either i or v may represent the state

3. No new state variable is defined by this element.

Components	Constitutive Equation	Energy Stored or Power Dissipated
Capacitor	$i = C\dfrac{dv}{dt}$	$E = \dfrac{1}{2}Cv^2$
Inductor	$v = L\dfrac{di}{dt}$	$E = \dfrac{1}{2}Li^2$
Resistor	$v = iR$	$P = \dfrac{v^2}{R}$ or $P = I^2R$

Note:

- Voltage is a natural output variable and current is a natural input variable for a capacitor.

- Current is a natural output variable; voltage is a natural input variable and voltage is a natural state variable for an inductor.

System Type System-Variables:	Mechanical	Electrical
Through-Variables	Force f	Current i
Across- Variables	Velocity v	Voltage v
System Parameters	m k b	C $1/L$ $1/R$

Variables: Across variable temperature (T) and through variable heat transfer rate (Q).
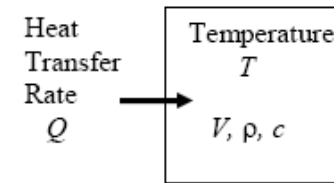
Thermal Capacitor (A-Type Element)

Consider control volume V of fluid with, density ρ, and specific heat c.

Constitutive Equation: Net heat transfer rate into the control volume $Q = \rho V c \dfrac{dT}{dt}$ ➔

$$C_t \frac{dT}{dt} = Q$$

$C_t = \rho v c$ = thermal capacitance of control volume

Heat Transfer Rate Q → | Temperature T / V, ρ, c

Observations:
Temperature T is state variable for thermal capacitor (from usual argument) ➔
"**A-Type Element**"
Heat transfer rate Q is natural input and temperature T is natural output for this element
This is a storage element (stores thermal energy)

Note: There is no thermal "inductor" like storage element with state variable Q .

Thermal Resistance (D-Type Element)

Three basic processes of heat transfer ➜ three different types of thermal resistance

Constitutive Relations

Conduction: $Q = \dfrac{kA}{\Delta x} T$

k = conductivity; A = area of cross section of the heat conduction element; Δx = length of heat conduction that has a temperature drop of T.

➜ Conductive resistance $R_k = \dfrac{\Delta x}{kA}$

Convection: $Q = h_c A T$

h_c = convection heat transfer coefficient; A = area of heat convection surface with temperature drop T

➜ Conductive resistance $R_c = \dfrac{1}{h_c A}$

Radiation: $Q = \sigma F_E F_A A (T_1^4 - T_2^4)$ ➜ a nonlinear thermal resistor

σ = Stefan-Boltzman constant
F_E = effective emmisivity of the radiation source (of temperature T_1)
F_A = shape factor of the radiation receiver (of temperature T_2)
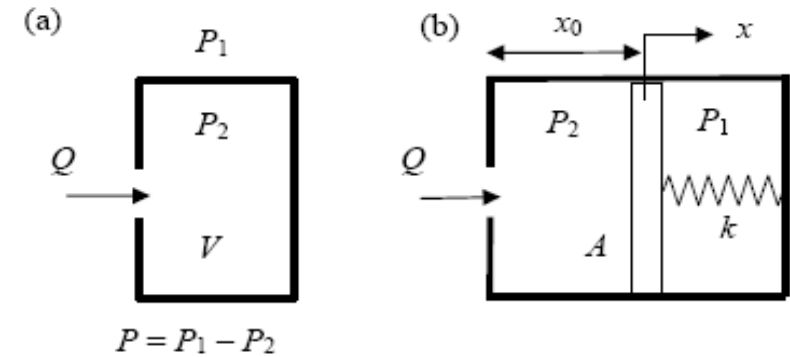A = effective surface area of the receiver.

Variables: Pressure (across variable) P and volume flow rate (through variable) Q

Fluid Capacitor (A-Type Element)

Constitutive Equation: $C_f \dfrac{dP}{dt} = Q$

Note 1: Stores potential energy (a "fluid spring")
Note 2: Pressure (across variable) is state variable
for fluid capacitor ➔ "**A-Type Element**"

(a)

P_1

P_2

Q

V

$P = P_1 - P_2$

(b)

x_0

x

P_2

P_1

Q

A

k

Three Types: Fluid compression; Flexible container; Gravity head

1a. For liquid control volume V of bulk modulus β: $C_{bulk} = \dfrac{V}{\beta}$

1b. For isothermal (constant temperature, slow-process) gas of volume V and pressure:

$C_{comp} = \dfrac{V}{P}$

(c)

1. For adiabatic (zero heat transfer, fast-process) gas: $C_{comp} = \dfrac{V}{kP}$

$k = \dfrac{c_p}{c_v}$ = ratio of specific heats at constant pressure and constant volume

P_1 A

h

P_2 Q

2. For incompressible fluid in a flexible vessel of area A and stiffness k: $C_{elastic} = \dfrac{A^2}{k}$

Note: For a fluid with bulk modulus, the equivalent capacitance $= C_{bulk} + C_{elastic}$.

3. For incompressible fluid column of area of cross-section A and density ρ: $C_{grav} = \dfrac{A}{\rho g}$
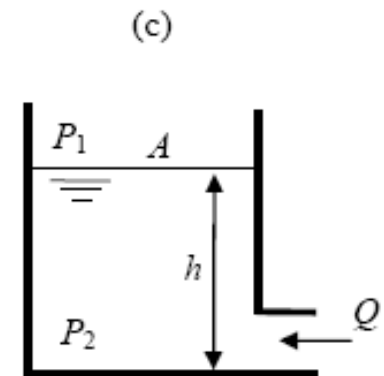
Fluid Inertor (T-Type Element)

Constitutive Equation: $I_f \dfrac{dQ}{dt} = P$

Note 1: Volume flow rate Q (through variable) is state variable for fluid inertor ➔ "**T-type Element**"

Note 2: It stores kinetic energy, unlike the mechanical T-type element (spring), which stores potential energy.
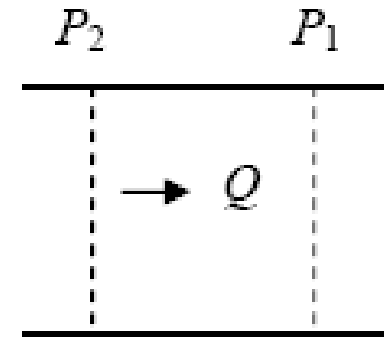
With uniform velocity distribution across A over length segment Δx :

Fluid inertance $I_f = \rho \dfrac{\Delta x}{A}$

For a non-uniform velocity distribution:

Fluid inertance $I_f = \alpha \rho \dfrac{\Delta x}{A}$ (correction factor α)

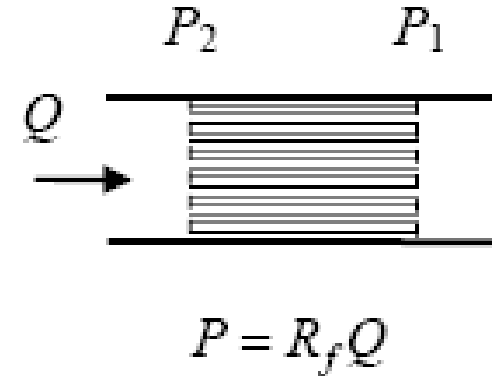For a pipe of circular cross-section with a parabolic velocity distribution, $\alpha = 2.0$

$P_2 \qquad P_1$

Q

$P = I_f \dfrac{dQ}{dt}$

Fluid Resistor (D-Type Element)

Constitutive Equation (Linear): $P = R_f Q$

Constitutive Equation (Nonlinear): $P = K_R Q^n$

(K_R and n are parameters of nonlinearity)

For Viscous Flow Through a Uniform Pipe:

(a) With circular cross-section of diameter d: $R_f = 128\,\mu \dfrac{\Delta x}{\pi d^4}$

(b) With rectangular cross-section of height $b \ll$ width w: $R_f = 12\mu \dfrac{\Delta x}{wb^3}$

**Note:** μ = absolute viscosity (or, dynamic viscosity); υ = kinematic viscosity with $\mu = \upsilon\rho$

System Type	Constitutive Relation for		
	Energy Storage Elements		Energy Dissipating Elements
	A-Type (Across) Element	T-Type (Through) Element	D-Type (Dissipative) Element
Translatory-Mechanical v = velocity f = force	Mass (Newton's 2nd Law) m = mass	Spring (Hooke's Law) k = stiffness	Viscous Damper b = damping constant
Electrical v = voltage i = current	Capacitor C = capacitance	Inductor L = inductance	Resistor R = resistance
Thermal T = temperature difference Q = heat transfer rate	Thermal Capacitor C_t = thermal capacitance	None	Thermal Resistor R_t = thermal resistance
Fluid P = pressure difference Q = volume flow rate	Fluid Capacitor C_f = fluid capacitance	Fluid Inertor I_f = inertance	Fluid Resistor R_f = fluid resistance

System Type	Through Variable	Across Variable
Hydraulic/Pneumatic	Flow Rate	Pressure
Electrical	Current	Voltage
Mechanical	Force	Velocity
Thermal	Heat Transfer	Temperature

Suspension of a car

$y(t)$

M_2

Ks B

$x(t)$

M_1

Kw

Road surface

$r(t)$
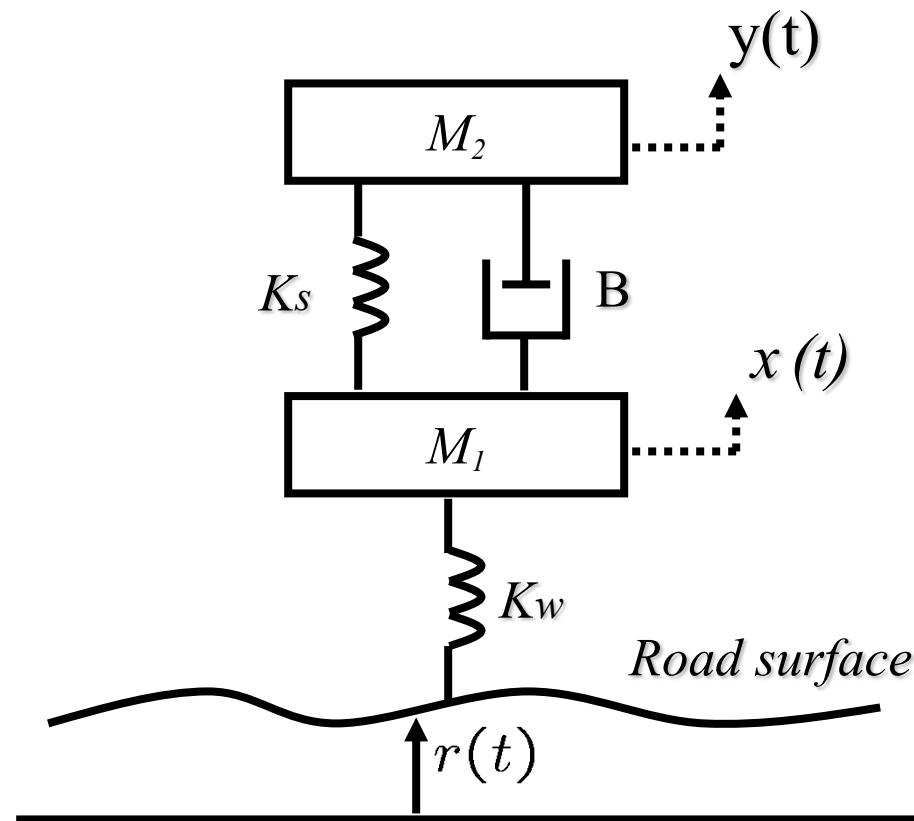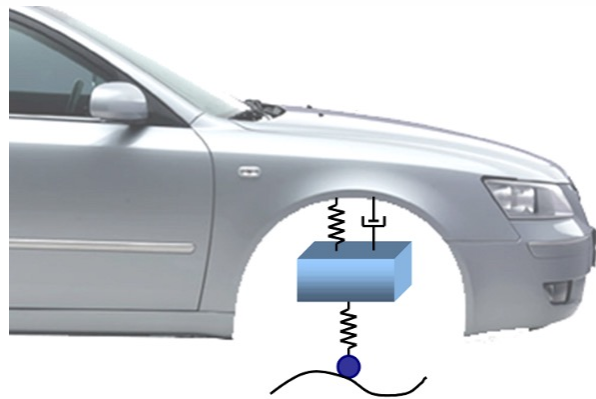
Electrical Circuit
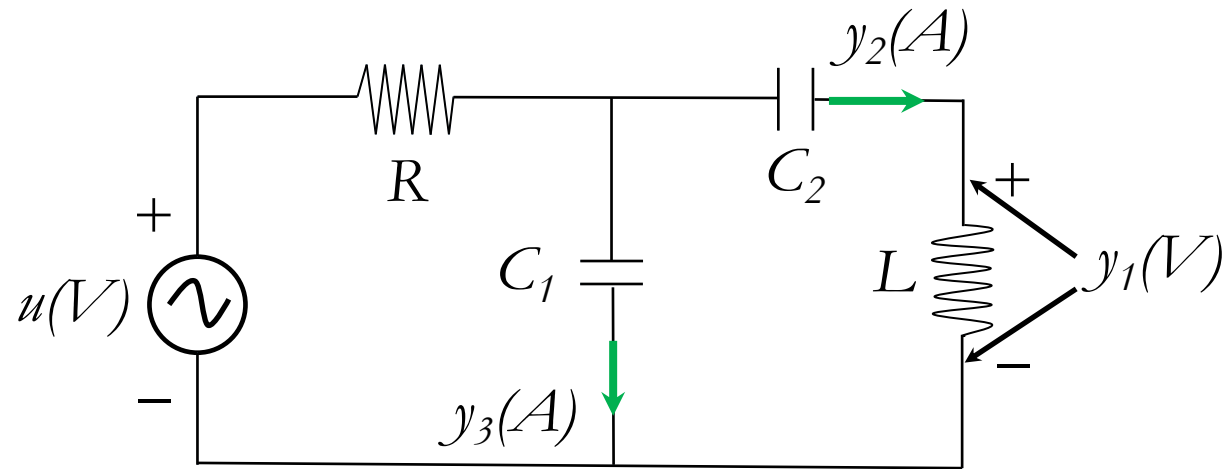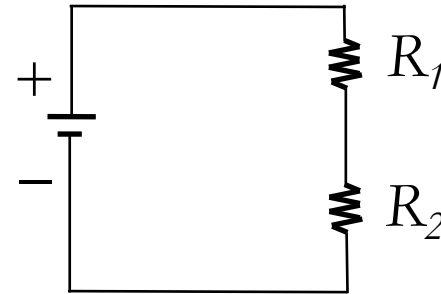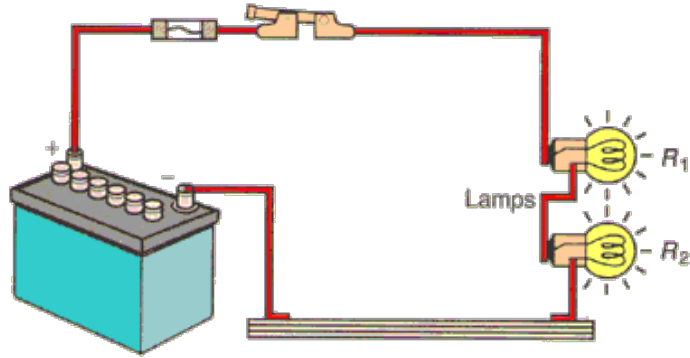
DC Motor (will discuss it in detail in later chapter)

$$ia(t) \quad Ra \quad La$$

$$\omega_m(t) \quad \textit{Output}$$
$$\theta_m(t)$$

$$ea(t) \qquad eb(t) \qquad \boldsymbol{J_m} \qquad T_L(t)$$

$$\textit{Input} \qquad\qquad T_m(t) \qquad \boldsymbol{B_m}$$

Armature circuit   *Mechanical load*

The End!!